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NOTE ON THE GENERAL SOLUTION OF THE TRANSFER PROCESSES IN FINITE CAPILLARY POROUS BODY

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In [1] it is shown that the temperature and moisture distributions in one-dimensional bodies (plate, cylinder and sphere) are linear combinations of the solutions of boundary-value problems of pure heat conduction if the boundary conditions are of the same type for both potentials. In the present study the same will be demonstrated for a finite homogeneous region of arbitrary geometry.

The process of drying is described by a Luikov's system of equations [2]

$$\frac{\partial\theta_1(M,Fo)}{\partial Fo} = \nabla^2\theta_1(M,Fo) - Ko^*\frac{\partial\theta_2(M,Fo)}{\partial Fo} \qquad (1)$$

$$\frac{\partial\theta_2(M,Fo)}{\partial Fo} = Lu\nabla^2\theta_2(M,Fo) - LuPn\nabla^2\theta_1(M,Fo). \qquad (2)$$

In above equations $\theta_1(M, Fo)$ denotes dimensionless temperature; $\theta_2(M, Fo)$, dimensionless mass-transfer potential; M, position of a point in finite region V; Fo, Fourier number; $Ko^* = \varepsilon Ko$; ε, phase change criterion; Ko, Kossovich number; Lu, Luikov number; Pn, Posnov number; ∇^2, the Laplacian [2].

The initial potentials are prescribed functions defined in

$$\theta_k(M,0) = f_k(M), \quad k = 1, 2. \qquad (3)$$

The boundary conditions are

$$A(N)\frac{\partial\theta_k(N,Fo)}{\partial n} + B(N)\theta_k(N,Fo) = \varphi_k(N,Fo), \qquad k = 1,2 \qquad (4)$$

where n is outward normal of S; S, boundary of V; $A(N)$ and $B(N)$, prescribed boundary coefficient functions defined on S; N, position of a point in the surface S; $\varphi_k(N, Fo)$, source functions on S:

The solution of the Sturm–Liouville problem

$$\nabla^2\psi_i(M) + \mu_i^2\psi_i(M) = 0 \qquad (5)$$

$$A(N)\frac{\partial\psi_i(N)}{\partial n} + B(N)\psi_i(N) = 0 \qquad (6)$$

is granted for known.

For the solution of problem (1)–(4) it is convenient to apply a three dimensional finite integral transform [3, 4].

$$\bar{\theta}_{ki}(Fo) = \int_V \psi_i(M)\,\theta_k(M,Fo)\,\mathrm{d}V. \qquad (7)$$

It follows from the orthogonality of the eigenfunctions that the inversion formula for (7) is

$$\theta(M, Fo) = \sum_{i=1}^{\infty} \frac{\psi_i(M)\tilde{\theta}_{ki}(Fo)}{\int_V \psi_i^2(M)\,\mathrm{d}V}. \tag{8}$$

By applying the transform (7) we take the integral transform of $\nabla^2\theta_k(M, Fo)$

$$\int_V \psi_i(M)\,\nabla^2\theta_k(M, Fo)\,\mathrm{d}V = \varphi_k(Fo) - \mu_i^2\tilde{\theta}_{k,i}(Fo) \tag{9}$$

where

$$\varphi_k(Fo) = \int_S \varphi_k(N, Fo)\frac{\psi_i(N) - \partial\psi_i(N)/\partial n}{A(N) + B(N)}\,\mathrm{d}S. \tag{10}$$

Multiplying the above equations (1) and (2) by $\psi_i(M)$ and integrating, after taking into account (7) and (10), one gets

$$\frac{\mathrm{d}\tilde{\theta}_{1,i}(Fo)}{\mathrm{d}Fo} = \psi_1(Fo) - \mu_i^2\tilde{\theta}_{1,i}(Fo) - Ko^*\frac{\mathrm{d}\tilde{\theta}_{2,i}(Fo)}{\mathrm{d}Fo} \tag{11}$$

$$\frac{\mathrm{d}\tilde{\theta}_{2,i}(Fo)}{\mathrm{d}Fo} = Lu[\varphi_2(Fo) - \mu_i^2\tilde{\theta}_{2,i}(Fo)] - LuPn[\varphi_1(Fo) - \mu_i^2\tilde{\theta}_{1,i}(Fo)]. \tag{12}$$

Using the Laplace Transform with parameter p we obtain

$$(p + \mu_i^2)\bar{\theta}_1(p) + pKo^*\bar{\theta}_2(p) = \tilde{\theta}_{1,i}(0) + Ko^*\tilde{\theta}_{2,i}(0) + \bar{\varphi}_1(p) \tag{13}$$

$$-LuPn\mu_i^2\bar{\theta}_1(p) + (p + Lu\mu_i^2)\bar{\theta}_2(p) = \tilde{\theta}_{2,i}(0) + Lu\bar{\varphi}_2(p) - Lu\bar{\varphi}_1(p) \tag{14}$$

where

$$\tilde{\theta}_{k,i}(0) = \int_V \psi_i(M)f_k(M)\,\mathrm{d}V \tag{15}$$

$$\bar{\varphi}_k(p) = \int_s \bar{\varphi}_k(N, p)\frac{\psi_i(N) - \partial\psi_i(N)/\partial n}{A(N) + B(N)}\,\mathrm{d}s. \tag{16}$$

The solution of the simultaneous equations (13) and (14) is

$$\bar{\theta}_1(p) = \frac{1}{(p + Lu\mu_i^2 v_1^2)(p + Lu\mu_i^2 v_2^2)}\{(p + Lu\mu_i^2)\tilde{\theta}_{1,i}(0) + Ko^*Lu\mu_i^2\tilde{\theta}_{2,i}(0) + Lu[(v_1^2 + v_2^2 - 1)p + \mu_i^2]\bar{\varphi}_1(p) - LuKo^*p\bar{\varphi}_2(p)\} \tag{17}$$

$$\bar{\theta}_2(p) = \frac{1}{(p + Lu\mu_i^2 v_1^2)(p + Lu\mu_i^2 v_2^2)}\{PnLu\mu_i^2\tilde{\theta}_{1,i}(0) + [p + (v_1^2 + v_2^2 - 1)Lu\mu_i^2]\tilde{\theta}_{2,i}(0) - PnLup\bar{\varphi}_1(p) + Lu(p + \mu_i^2)\bar{\varphi}_2(p)\} \tag{18}$$

where

$$v_j^2 = \tfrac{1}{2}\left\{1 + Ko^*Pn + \frac{1}{Lu} + (-1)^j\sqrt{\left[\left(1 + Ko^*Pn + \frac{1}{Lu}\right)^2 - \frac{4}{Lu}\right]}\right\}. \tag{19}$$

Using the inverse Laplace transform we then obtain

$$\tilde{\theta}_{1,i}(Fo) = \sum_{k=1}^{2}\sum_{j=1}^{2} A_{kj}\exp(-\mu_i^2 v_j^2 LuFo)[\tilde{\theta}_{k,i}(0) + \int_0^{Fo}\exp(\mu_i^2 v_j^2 LuFo)\varphi_k(Fo)\,\mathrm{d}(v_j^2 LuFo)] \tag{20}$$

$$\tilde{\theta}_{2,i}(Fo) = \sum_{k=1}^{2}\sum_{j=1}^{2} B_{kj}\exp(-\mu_i^2 v_j^2 LuFo)[\tilde{\theta}_{k,i}(0) + \int_0^{Fo}\exp(\mu_i^2 v_j^2 LuFo)\varphi_k(Fo)\,\mathrm{d}(v_j^2 LuFo)] \tag{21}$$

$$A_{k,j} = (-1)^j\frac{(k-1)Ko^* - (k-2)(1 - v_j^2)}{v_1^2 - v_2^2} \tag{22}$$

$$B_{kj} = (-1)^j\frac{(k-1)(v_{3-j}^2 - 1) - (k-2)Pn}{v_1^2 - v_2^2}. \tag{23}$$

Substituting (20) and (21) in the inversion formula (8), the desired solutions are obtained as follows

$$\theta_1(M, Fo) = \sum_{k=1}^{2}\sum_{j=1}^{2} A_{kj}\theta_{kj}(M, Fo) \tag{24}$$

$$\theta_2(M, Fo) = \sum_{k=1}^{2}\sum_{j=1}^{2} B_{kj}\theta_{kj}(M, Fo) \tag{25}$$

where

$$\theta_{kj}(M, Fo) = \sum_{i=1}^{\infty}\frac{\psi_i(M)}{\int_V\psi_i^2(M)\,\mathrm{d}V}\exp(-\mu_i^2 v_j^2 LuFo)\left\{\int_V\psi_i(M)f_k(M)\,\mathrm{d}V + \int_0^{Fo}\exp(\mu_i^2 v_j^2 LuFo)\left[\int_S\varphi_k(N, Fo)\frac{\psi_i(N) - \partial\psi_i(N)/\partial n}{A(N) + B(N)}\,\mathrm{d}S\right]\mathrm{d}(v_j^2 LuFo)\right\}. \tag{26}$$

If $B(N) = 0$ then $\psi_0 = \text{const}$ and $\mu_0 = 0$ are also eigenfunction and eigenvalue of the Sturm–Liouville problem (5) and (6). Therefore, in this case an additional term, corresponding to the zero-eigenvalue appears and (26) takes the form

$$\theta_{kj}^*(M, Fo) = \frac{1}{V}\left\{\int_V f_k(M)\,\mathrm{d}V + \int_0^{Fo}\int_S\frac{\psi_i(N, Fo)}{A(N)}\,\mathrm{d}S\,\mathrm{d}(v_j^2 LuFo)\right\} + \sum_{i=1}^{\infty}\frac{\psi_i(M)}{\int_V\psi_i^2(M)\,\mathrm{d}V}\exp(-\mu_i^2 v_j^2 LuFo)\left\{\int_V\psi_i(M)f_k(M)\,\mathrm{d}V\right.$$

$$+\int_0^{Fo}\exp(\mu_i^2 v_j^2 LuFo)\int_S \varphi_k(N, Fo)\,\frac{\psi_i(N)}{A(N)}\,\mathrm{d}S\,\mathrm{d}(v_j^2 LuFo)\Big\}. \quad (27)$$

Formulae (26) and (27) coincide completely with those of pure heat conduction if we make the substitution $\tau = v_j^2 LuFo$. Therefore nomogramm 2 given by Krischer in [5], page 8, supposing the time scale is 4 times less, becomes identical with the generally known nomogramm for plate cooling (see e.g. [6], p. 62, Fig. 29).

Consequently the decisions (24) and (25) represent linear combinations of the solution of boundary-value problems of pure heat conduction. They are easy to apply taking the numerical values for A_{kj} and B_{kj} depending on the Luikov's number, Pn and Ko^*Pn given in Tables in [1].

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USE OF COATINGS OF LOW THERMAL CONDUCTIVITY TO IMPROVE FINS USED IN BOILING LIQUIDS

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NOMENCLATURE

- B, defined as $h\Delta T_2/k_1 w$;
- h, boiling heat transfer coefficient [Btu/h ft^2 °F];
- k_1, thermal conductivity of prime fin material [Btu/h ft °F];
- k_2, thermal conductivity of coating [Btu/h ft °F];
- Q, heat transfer rate [Btu/h];
- r, local radius [ft];
- r_B, radius at base of fin [ft];
- R, radius at tip of fin [ft];
- ΔT_1, temperature excess above liquid boiling point of prime fin material (a function of r) [°R];
- ΔT_2, temperature excess of wetted surface of coating (not a function of r) [°R];
- ΔT_B, temperature excess at the base of the prime fin material [°R];
- w, half-thickness of fin [ft]. (Conversion of units: 1 W = 3·414 Btu/h).

BACKGROUND

The first use of an insulating coating to improve boiling heat transfer was reported by Cowley *et al.* [1]. They showed that if the surface of a solid is so hot as to cause film boiling to an ambient liquid, one may achieve a higher heat transfer rate by applying a prescribed thickness of an insulating material. The coating thickness must be such as to result in a low enough surface temperature to cause nucleate boiling. Nucleate boiling heat transfer coefficients are often one or two orders of magnitude greater than film boiling coefficients, thus the gain in surface coefficient may offset the penalty of thermal resistance of the coating. Cowley tried coatings of varnish, vaseline, asbestos, sodium silicate, and other materials on five metals in three liquids and obtained improvements in the heat flux by as much as 500 per cent.

Recently, Rubin *et al.* [2] rediscovered the use of insulation to improve boiling heat fluxes. They reported tests with a single copper spine, 0·79 in. long by 0·4 in. dia., sheathed